機械與機電博士班資格考試題庫

科目：工程數學

1.參考書本:

D. G. Zill, W. S. Wright, Advanced Engineering Mathematics 5^{th} ed., Jones & Bartlett Learning, 2014.

2.考試章節:

(a) First-order differential equations

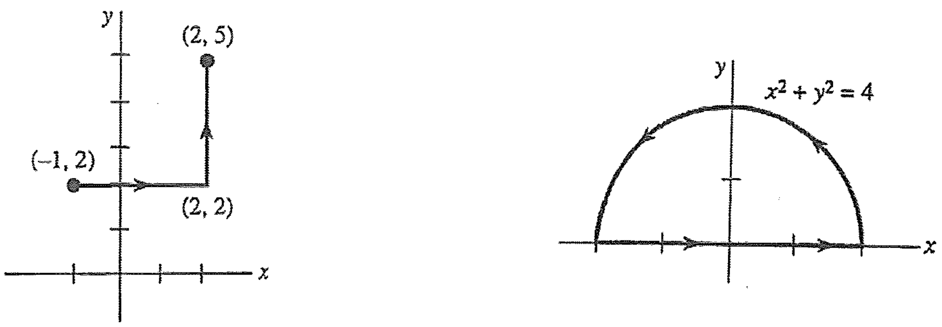
(b) Higher-order differential equations

(c) The Laplace transform

(d) Matrices

(e) Vectors Calculus

(f) Fourier Series



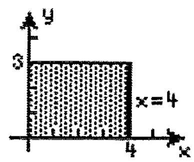
(a) (b)

(a) Evaluate $\int_c (2x + y)\,dx + xy\,dy$ on the given curve C between (-1, 2) and (2, 5)

(b) Evaluate $\oint_c (x^2 + y^2)\,dx - 2xy\,dy$ on the given closed C

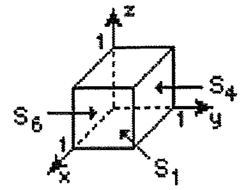
Find the center of mass of the lamina that has the given shape and density.

$x = 0, x = 4, y = 0, y = 3;\ \rho(x, y) = xy$



Verify the divergence theorem.

$\mathbf{F} = xy\,\mathbf{i} + yz\,\mathbf{j} + xz\,\mathbf{k}$ D the region bounded by the unit cube defined by $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$



Find the eigenvalues and eigenvectors of the given nonsingular matrix **A**. Then without finding $\mathbf{A}^{-1}$, find its eigenvalues and corresponding eigenvectors.

1. $\mathbf{A} = \begin{pmatrix} 4 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 5 \end{pmatrix}$

2. $\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$

Use "Laplace transform" to solve the initial values problem (IVP):

1. $y' - y = 2cos5t,\ y(0) = 0$

2. $y'' - 4y' = 6e^{3t} - 3e^{-t},\ y(0) = 1,\ y'(0) = -1$

3. $y'' - 2y' + 5y = 1 + t,\ y(0) = 0,\ y'(0) = 4$

4. $y' + y = f(t), y(0) = 0,\ where\ f(t) = \begin{cases} 1, & 0 \le t < 1 \\ -1, & t \ge 1 \end{cases}$

5. $y'' + 4y = sint u(t - 2\pi),\ y(0) = 1,\ y'(0) = 0$

6. $f(t) + 2\int_0^t f(\tau)\cos(t-\tau)\,d\tau = 4e^{-t} + sint$

Solve the given differential equation:

1. $y'' + 2y' = 2x + 5 - e^{-2x}$

2. $y'' + 4y = 3sin2x$

3. $y'' + 2y' + y = sinx + 3cos2x$

4. $x^2y'' + 3xy' + 13y = 4 + 3x$

5. $x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + lnx^3$

Solve the given initial-value problem (IVP):

1. $xy' + y = e^x, \ y(1) = 2$

2. $y\frac{dx}{dy} - x = 2y^2, \ y(1) = 5$

3. $(x + y)^2 dx + (2xy + x^2 - 1)dy = 0, \ y(1) = 1$

4. $(4y + 2t - 5)dt + (6y + 4t - 1)dy = 0, \ y(-1) = 1$

Find the Fourier series of f on the given interval:

1. $f(x) = \begin{cases} 1, & -1 < x < 0 \\ x, & 0 \le x < 1 \end{cases}$

2. $f(x) = \begin{cases} 0, & -1 < x < 0 \\ x, & 0 \le x < 1 \end{cases}$

3. $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \le x < \pi \end{cases}$

4. $f(x) = \begin{cases} \pi^2, & -\pi < x < 0 \\ \pi^2 - x^2, & 0 \le x < \pi \end{cases}$

Find the half-range cosine and sine expansions:

1. $f(x) = \begin{cases} 1, & -1 < x < \frac{1}{2} \\ 0, & \frac{1}{2} \le x < 1 \end{cases}$

2. $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$

3. $f(x) = x^2 + x,\ 0 < x < 1$