

1. Please explain the following terms. 30%

- (1) Centro
- (2) Simple curve
- (3) Regular simple curve
- (4) Singular point
- (5) Mechanisms

2. The surface $z = f(x, y)$ is said to have “Rulings” if through the point $p_0 = f(x_0, y_0, z_0)$ there is a straight line segment all of whose points are on the surface. This happens if through the point $(x_0, y_0, 0)$ in the xy -plane there is a line of points $(x_0 + h, y_0 + k, 0)$ such that along this line dz and Δz are equal. This happens if and only if

$$f(x_0 + h, y_0 + k) - f(x_0, y_0) = \left(\frac{\partial f}{\partial x}\right)_0 h + \left(\frac{\partial f}{\partial y}\right)_0 k \quad \text{when } h = \Delta x = dx \quad \text{and} \quad k = \Delta y = dy. \text{ Show}$$

that the surface $\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2} = 1$ is ruled surface or not for $h = k$. 30%

3. A position vector \vec{r} in coordinate systems is expressed as $\vec{r}(\alpha) = a \cos \alpha \vec{i} + a \sin \alpha \vec{j} + c \alpha \vec{k}$. Please determine (a) Unit tangent vector $\vec{t}(s)$ (b) Unit normal vector $\vec{n}(s)$ (c) Curvature κ (d) Unit binormal vector $\vec{b}(s)$. 40% (Hint: Where s is arc length, $ds^2 = d\vec{r} \cdot d\vec{r}$)